Recall the definitions of ellipse and hyperbola:

An ellipse is the set of all points in a plane such that the sum of their distances to two fixed points is a constant. A hyperbola is the set of all points in a plane such that the absolute value of the difference of their distances to two fixed points, the foci, is a constant.

The ellipse $4x^2 + 25y^2 = 100$ and the hyperbola $4x^2 - 25y^2 = 100$ are graphed on the right.

1. Tell the coordinates of the center and the endpoints of the major and minor axes of the ellipse.

2. a. Using dashed line segments draw an auxiliary rectangle with vertices $(5, 2), (5, -2), (-5, 2),$ and $(-5, -2).$ Also using dashed lines, draw two diagonal lines that pass through the center and vertices of the rectangle and extend to the edges of the grid.

b. What relationships do the rectangle and lines have to the ellipse and hyperbola?

c. Why are dashed lines used when sketching the rectangle and diagonals of the rectangle?

The transverse axis of a hyperbola has endpoints on the hyperbola. The center of a hyperbola is the midpoint of the transverse axis. The foci are on the line containing the transverse axis. The conjugate axis of the hyperbola is the line segment perpendicular to the transverse axis passing through the center of the hyperbola. The hyperbola has asymptotes, lines which the branches of the hyperbola approach. The asymptotes contain the center of the hyperbola and pass through the vertices of the auxiliary rectangle.
3. Complete the table below. The first row has been done for you using the hyperbola \( \frac{x^2}{49} - \frac{y^2}{9} = 1 \) as an example.

<table>
<thead>
<tr>
<th>Graph</th>
<th>Equation</th>
<th>Length, Endpoints, and Orientation of Transverse Axis</th>
<th>Length, Endpoints, and Orientation of Conjugate Axis</th>
<th>Equations of Asymptotes</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph 1" /></td>
<td>( \frac{x^2}{49} - \frac{y^2}{9} = 1 )</td>
<td>14 units (7, 0), (−7, 0) horizontal</td>
<td>6 units (0, 3), (0, −3) vertical</td>
<td>( y^2 = \frac{9}{49}x^2 ) ( y = \pm \frac{3}{7}x )</td>
</tr>
<tr>
<td><img src="image2.png" alt="Graph 2" /></td>
<td>( \frac{x^2}{4} - \frac{y^2}{25} = 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image3.png" alt="Graph 3" /></td>
<td>( \frac{y^2}{9} - \frac{x^2}{16} = 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td><img src="image4.png" alt="Graph 4" /></td>
<td>( \frac{y^2}{49} - \frac{x^2}{36} = 1 )</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
ACTIVITY 7.3 continued

Hyperbolas
What’s the Difference?

SUGGESTED LEARNING STRATEGIES: Look for a Pattern, Quickwrite, Notetaking, Vocabulary Organizer, Create Representations

4. How do the equations of the asymptotes relate to the equation of the hyperbola?

5. How can the direction in which the branches of the hyperbola open be determined by the equation?

The standard form of a hyperbola is \(\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1\), when the transverse axis is horizontal. The standard form of a hyperbola is \(\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1\) when the transverse axis is vertical. The endpoints of the transverse axis are the vertices of the branches, and are located \(a\) units from the center of the hyperbola that is located at the point \((h, k)\). The equations of the asymptotes are found by setting the quadratic terms equal to each other and solving for \(y\).

EXAMPLE 1
Sketch the hyperbola \(\frac{(x-1)^2}{16} - \frac{y^2}{49} = 1\). Tell the coordinates of the center and the vertices, and give the equations of the asymptotes

- The positive term is \(\frac{(x-1)^2}{16}\), so the transverse axis is horizontal.
- Since \(a^2\) is 16, then \(a = 4\) and the transverse axis is 8 units long.
- The center is \((1, 0)\).
- The vertices on the transverse axis are 4 units from the center: \((-3, 0)\) and \((5, 0)\).
- Setting \(\frac{(x-1)^2}{16} = \frac{y^2}{49}\) and solving for \(y\) gives the equations of the asymptotes.
  
  \[y^2 = \frac{49(x-1)^2}{16} \rightarrow y = \pm \frac{7(x-1)}{4}\]
EXAMPLE 2
Sketch the hyperbola \( \frac{(y + 3)^2}{25} - \frac{(x + 4)^2}{121} = 1 \). Tell the coordinates of the center and the vertices, and give the equations of the asymptotes.

- The positive term is \( \frac{(y + 3)^2}{25} \), so the transverse axis is vertical.
- Since \( a^2 \) is 25, then \( a = 5 \) and the transverse axis is 10 units long.
- The center is \((-4, -3)\).
- The vertices on the transverse axis are 5 units from the center: \((-4, 2)\) and \((-4, -8)\).
- Setting \( \frac{(y + 3)^2}{25} = \frac{(x + 4)^2}{121} \) and solving for \( y \) gives the equations of the asymptotes:
  \[ (y + 3)^2 = \frac{25(x + 4)^2}{121} \rightarrow (y + 3) = \pm \frac{5(x + 4)}{11} \rightarrow y = -3 \pm \frac{5(x + 4)}{11} \]

TRY THESE A
Write your answers on notebook or grid paper. Show your work. Sketch each hyperbola. Tell the coordinates of the center, label the vertices and give the equations of the asymptotes.

\[ a. \quad \frac{x^2}{100} - \frac{y^2}{49} = 1 \quad b. \quad \frac{y^2}{9} - \frac{x^2}{64} = 1 \quad c. \quad \frac{x^2}{16} - \frac{(y + 4)^2}{36} = 1 \]

\[ d. \quad \frac{(x + 2)^2}{25} - \frac{(y - 3)^2}{9} = 1 \]

6. \( \frac{x^2}{a^2} - \frac{y^2}{b^2} = 1 \) is a hyperbola centered at the origin. Find each item.

- a. the direction of the transverse axis
- b. the length and endpoints of the transverse axis
- c. the length of the conjugate axis
- d. the equation of the asymptotes
7. Complete the table below using the information given.

<table>
<thead>
<tr>
<th>Hyperbola</th>
<th>Center</th>
<th>Length and Orientation of Transverse Axis</th>
<th>Length and Orientation of Conjugate Axis</th>
<th>Equation of Hyperbola</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Graph" /></td>
<td>(0, 0)</td>
<td>8 units vertical</td>
<td>4 units horizontal</td>
<td></td>
</tr>
<tr>
<td><img src="image2.png" alt="Graph" /></td>
<td>(−1, 3)</td>
<td>12 units horizontal</td>
<td>6 units vertical</td>
<td></td>
</tr>
</tbody>
</table>
8. Write the equation and graph the hyperbola described.

a. center \((-1, 4)\), transverse axis 6 units, vertical conjugate axis 8 units

b. asymptotes \(y = \pm \frac{3}{4}x\), vertices \((4, 0), (-4, 0)\)
Hyperbolas

What’s the Difference?

SUGGESTED LEARNING STRATEGIES: Create Representations

The foci of a hyperbola are located on the transverse axis $c$ units from the center. The values $a$, $b$, and $c$ are related by the equation $c^2 = a^2 + b^2$.

9. Graph each hyperbola and label the foci with their coordinates.

a. $\frac{x^2}{81} - \frac{y^2}{25} = 1$

b. $\frac{(y + 2)^2}{4} - \frac{x^2}{25} = 1$
CHECK YOUR UNDERSTANDING

Write your answers on notebook or grid paper. Show your work.

For each hyperbola in Questions 1–5:

a. Give the coordinates of the center.
b. Tell the direction of the transverse axis.
c. Write the equations of the asymptotes.
d. Sketch the hyperbola and label the endpoints of the transverse axis.

1. \( \frac{x^2}{16} - \frac{y^2}{49} = 1 \)
2. \( \frac{y^2}{81} - \frac{x^2}{25} = 1 \)
3. \( \frac{(x + 5)^2}{9} - \frac{(y + 2)^2}{4} = 1 \)
4. \( \frac{(x - 2)^2}{1} - \frac{(y + 3)^2}{64} = 1 \)
5. \( \frac{(y - 2)^2}{100} - \frac{(x - 5)^2}{4} = 1 \)

For each hyperbola in Questions 6–7:

a. Give the coordinates of the center.
b. Tell the direction and length of the transverse axis.
c. Write the equations of the asymptotes.
d. Write the equation of the hyperbola.

6. 

7. 

8. **MATHEMATICAL REFLECTION** How do the asymptotes of a hyperbola help you graph the hyperbola?